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Exam II: MTH 111, Spring 2018

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Points = $\frac{62}{62}$

QUESTION 1. (12 points) Find y' and DO NOT SIMPLIFY

(i) $y = 4e^{(2x^2-4x)} + 2x - 5$

$y' = 4e^{(2x^2-4x)} \cdot (4x-4) + 2$

(ii) $y = (5x^2+3x)\sqrt{5x+10}$

$y = (5x^2+3)(5x+10)^{1/2}$

$y' = \left[(5x^2+3) \cdot \frac{1}{2}(5x+10)^{-1/2} \cdot 5\right] + \left[(5x+10)^{1/2} \cdot (10x)\right]$

(iii) $y = ln[(2x^5+4x^3-3x)(2x+7)^5]$

$y = \ln(2x^5+4x^3-3x) + \ln(2x+7)^5$

$y' = \frac{10x^4+12x^2-3}{2x^5+4x^3-3x} + \frac{10}{2x+7}$

(iv) $y = 3(e^{(3x+2)} + 7x^4 + 5x + 2)^4$

$y' = 12(e^{(3x+2)} + 7x^4 + 5x + 2)^3 \cdot (3e^{(3x+2)} + 28x^3 + 5)$

Haya

QUESTION 2. (i) **(6 points)** Does the line $L: x = 2t+1, y = 5t-1, z = -2t+3$ lie entirely inside the plane $x + 2y + z = 23$? If not, does it intersect the plane? If yes, then find the intersection point.

$L: \left.\begin{array}{l} x = 2t+1 \\ y = 5t-1 \\ z = -2t+3 \end{array}\right\} t \in \mathbb{R}$

$P \Rightarrow x + 2y + z = 23$

it doesnt lie intierly on the plane but intersects it at $(5.2, 9.5, -1.2)$

$P(L) \Rightarrow (2t+1) + 2(5t-1) + (-2t+3) = 23 \leftarrow 1$

$\cancel{2t} + 1 + 10t - 2 - \cancel{2t} + 3 = 23$

$10t + 2 = 23$

$10t = 21$

$t = 21/10$

$= 2.1$

$x = 2(2.1) + 1 = 5.2$

$2) \rightarrow y = 5(2.1) - 1 = 9.5$

$z = -2(2.1) + 3 = -1.2$

$Q = (5.2, 9.5, -1.2)$

the point of intersection is $(5.2, 9.5, -1.2) \leftarrow 3$

(ii) **(4 points)** Given $N = \langle -2, 3, 2 \rangle$ is perpendicular to the plane P and the point $(-1, 4, 2)$ lies inside the plane P. Find the equation of the plane P.

$N_x(x - P_x) + N_y(y - P_y) + N_z(z - P_z) = 0$

$-2(x+1) + 3(y-4) + 2(z-2) = 0 \Leftrightarrow$ plane.

(iii) **(4 points)** Find the distance between $Q = (10, 10, 33)$ and the plane $P: -2x + 2y - 5z = 21$.

$D = \dfrac{|P(Q)|}{|N|}$ $\quad \dfrac{|-2(10) + 2(10) - 5(33) - 21|}{\sqrt{4+4+25}} = \dfrac{186}{\sqrt{33}}$ units.

(iv) **(6 points)** The two planes $P_1: x + 4y + z = 10$ and $P_2: -x + 2y - z = 8$ intersects in a line L. Find a parametric equations of L.

$N_1 \times N_2 = D$

$N_1 = \langle 1, 4, 1 \rangle$

$N_2 = \langle -1, 2, -1 \rangle$

$\begin{array}{ccc} i & j & k \\ 1 & 4 & 1 \\ -1 & 2 & -1 \end{array}$

$(-4-2)i - (-1+1)j + (2+4)k$

$1 \rightarrow D = \langle -6, 0, 6 \rangle$

let $z = 0$

$-x + 2y = 8$

$x + 4y = 10$

$x = -2$

$y = 3$

$z = 0$

$2 \rightarrow Q = (-2, 3, 0)$

$D = \langle -6, 0, 6 \rangle$

$3 \rightarrow L: \left.\begin{array}{l} x = -6t - 2 \\ y = 3 \\ z = 6t \end{array}\right\} t \in \mathbb{R}$

(v) **(4 points)** Can we draw the vector $V = \langle 1, -2, -6 \rangle$ inside $P: 5x + 7y - 3z = 19$? explain

$V \cdot N$ must $= 0$

$\langle 1, -2, -6 \rangle$
$\langle 5, 7, -3 \rangle$

$V \cdot N = 5 + -14 + 18 = 9$

$\therefore$ No you cannot draw V on the plane because the Dot product of V and the Normal is not 0

QUESTION 3. (7 points) Let $f(x) = e^{(x^2+2x+1)} + 3$.

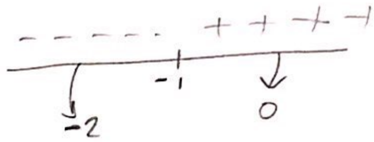
(i) For what values of x does $f(x)$ increase?

$f'(x) = e^{(x^2+2x+1)} \cdot (2x+2)$

$e^{(x^2+2x+1)} \cdot (2x+2) = 0$

$/(2x+2) \quad /(2x+2)$

Now e^{x^2 + 2x + 1} = 0 or (2x + 2) = 0
Note e^{anything} is never zero. Hence
2x + 2 = 0. Thus x = -1.



$\leftarrow$ 2

$f'(-2) = -$

$f'(0) = +$

$\leftarrow$ 3

$\therefore$ $f(x)$ increases from

$(-1, +\infty)$

(ii) For what values of x does $f(x)$ decrease?

$f(x)$ is decreasing from $(-\infty, -1)$

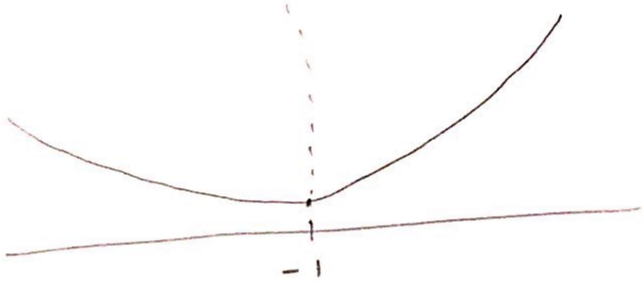
(iii) Find all local minimum, maximum points of $f(x)$ (just find the x-values where local min. and local max exist).

[No local or absolute maximum]

[local and absolute minimum at $x = -1$
point $(-1, 4)$]

$f(-1) = 4$

(iv) Roughly, sketch the graph of $f(x)$.



QUESTION 4. (5 points) Let $f(x) = ln(5x-4)+4$. Find the equation of the tangent line to the curve of $f(x)$ at $x = 1$.

$f(1) = \ln(5-4)+4$
$= 4$
point $= (1, 4)$

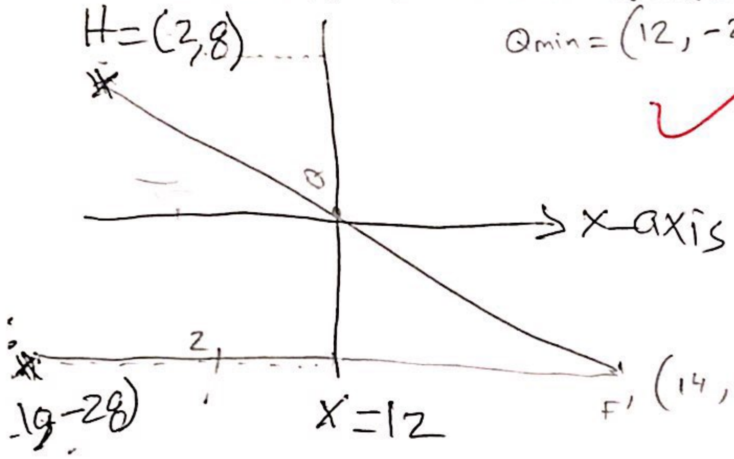
$f'(x) = \dfrac{5}{5x-4}$

1 → $f'(1) = m = \dfrac{5}{1}$

$y = 5x + b$
$4 = 5(1) + b$
2 → $b = 4 - 5$
$= -1$

3 → $\boxed{y = 5x - 1}$ is the equation of the tangent line.

QUESTION 5. (7 points) Given H and F. Find a point Q on the line $x = 12$ such that $|HQ| + |FQ|$ is minimum.



Qmin $= (12, -22)$

($H = (2,8)$,
$F = (10,-28)$)

$m = \dfrac{-28-8}{14-2} = -3$

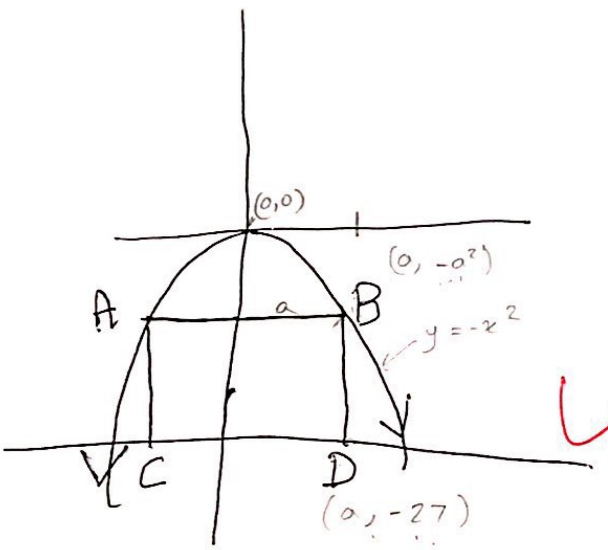
1 → $y = -3x + b$
$8 = -3(2) + b$
$b = 8 + 6$
$= 14$
$y = -3x + 14$

$|AC|$

$Q = (12, y)$
$y = -3(12) + 14$
$= -22$
$Q_{min} = (12, -22)$

QUESTION 6. (7 points) Consider the following picture. Find $|AB|$ and $|\cancel{CD}|$ so that the area is MAXIMUM.

The curve is $y = -x^2$,
the line $y = -27$



$A = |BD||AB|$

1 → $A = [-a^2 - (-27)] \cdot 2a$
$A = (-a^2 + 27) \cdot 2a$
$A = -2a^3 + 54a$
$A' = -6a^2 + 54$
$-6a^2 + 54 = 0$
$-6a^2 = -54$
$a^2 = -54/-6$
$a = \sqrt{9}$
$a = +3$

$A'' = -12a$
2 → $A''(3) = -$ ∴ max.

$A = (-3^2 + 27)(6)$
$= 108$ units^2

$|AB| = 2a$
$= 6$ units

$|BD| = (-a^2 + 27)$
$= 18$ units

Faculty information

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